

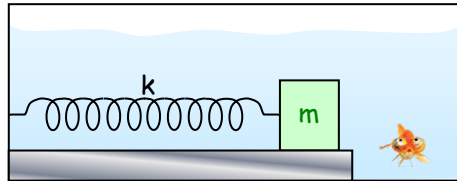
**TM5 3-2** Altered: Allow the motion of a 100 g mass attached to a  $k = 10^4$  dyne/cm initially displaced 3 cm from the equilibrium point and released from rest to take place in a resisting medium. After oscillating for  $10\tau_S$  (10 system periods), the maximum amplitude decreases to half the initial value. Find the

**a)** natural frequencies & period ( $\omega_N$ ,  $f_N$ , &  $\tau_N$ ) and

**b)** damping parameter  $\beta$ , and system frequencies and period ( $\omega_S$ ,  $f_S$ , &  $\tau_S$ ).

Compare the with the undamped & damped values.  $1 \text{ d} = 1 \text{ g-cm/s}^2 = 10^{-5} \text{ N}$  but don't convert! **Stay in cgs!**

$$\begin{aligned} m &= 100 \text{ g} \\ k &= 1.00\text{E}+04 \text{ d/cm} \\ A_0 &= 3 \text{ cm} \\ A(n\tau_S) &= 0.5 A_0 \\ n &= 10 \\ R &= A_0/A(10\tau_S) = 1/(1/2) = 2 \end{aligned}$$



$$\text{a) } \omega_N = \frac{\sqrt{k}}{\sqrt{m}} = 10.00 \text{ s}^{-1}$$

$$v_N = \frac{\omega_N}{2\pi} = 1.591549 \text{ Hz}$$

$$\tau_N = \frac{1}{v_N} = 0.628319 \text{ s}$$

$$\beta^2 = \frac{[\ln(R)]^2}{(2\pi n)^2} [(\omega_n)^2 - b^2] = \frac{0.693147}{1974.614} = 0.000351 \text{ s}^{-1}$$

$$\beta = 0.018736 \text{ s}^{-1} \quad \beta^2/[2(\omega_N)^2] = 1.755\text{E-}06$$

$$\text{b) } \omega_S = \sqrt{[(\omega_n)^2 - \beta^2]} = 9.999982 \text{ s}^{-1}$$

$$v_S = \frac{\omega_S}{2\pi} = 1.591547 \text{ Hz}$$

$$v_S/v_N = 99.99982\%$$

or

$$v_S = v_N\{1 - \beta^2/[2(\omega_N)^2]\} = v_N\{1 - 1.755 \times 10^{-6}\} \text{ TINY Difference!!}$$

$$\tau_N = \frac{1}{v_S} = 0.628320 \text{ s}$$

**The damping shortens the period by 0.00001 seconds!!**